Poza, D., Villafáñez, F., and Pajares J.

Abstract In this paper we replicate the model by Axtell et al. (2000), a game where two agents ask for proportions of the same pie. After simulating the same scenarios, we get the same results, both in the cases of one-agent and two-agent types (tag model). Once we know the model has been properly replicated, we go one step further, by analyzing the influence in the observed behavior of the 'rational' decision rule and of the matrix payoff. First, we change the agent's decision rule, so that agents could decide playing a heuristic which is not so 'rational' as the original rule. We also evaluate how dependant are results on the selected payoff matrix. We conclude that both the decisions rules and the payoff matrix could affect how and when the equilibrium and the segregation emerge in the system. This is particularly interesting for the tag model, as it is related to the role of group recognition in economic decisions.

Key words: Replication, Agent-based modelling, Validation, Verification.

1 Introduction

Agent-based modelling has become an extremely useful methodology. Restrictions in time and availability (among others) make it difficult to involve humans in experiments.

If both social sciences and economics are experimental sciences, they need a laboratory (Lopez-Paredes et al., 2002). By means of bottom-up models, social scientists have been able to analyze emergent social phenomena beyond the traditional simulation and experimental techniques. From micro behaviours and interactions



D. J. Poza, F. A. Villafáñez, and J. Pajares.

INSISOC, Valladolid University, Paseo del Cauce s/n 47011 Valladolid, Spain; e-mail: djpoza@gmail.com, villafafelix@yahoo.es, pajares@eis.uva.es

among agents, we have been able to build stylized models explaining some of the relevant macro-observed facts.

The pioneering works by Schelling and Axelrod showed us how computational sciences might help social scientists to develop models based on real assumptions about the behaviour of economic agents. However, we had to wait for several decades to put those pioneering works to the test. Thus, the research by Schelling (1971) was intensively extended by Epstein & Axtell (1996), who built up a real Universe (Sugarscape) by means of simple rules. More recently, Galan Izquierdo (2005) discussed the meta-norm models by Axelrod (1986).

The efforts for replicating previous published models have grown during the recent years. Replication contributes to improve the reliability of the results and understanding of the system, as Sansores and Pavón. (2005) stated. However, model replicating is a very tough task, as it was showed by Axelrod et al. (1997) and Edmonds Hales (2003). More recently, Wilensky & Rand (2007) proposed some interesting recommendations for improving diffusion and rigour in multiagent simulations. Anyway, replication is always the first step to improve and extending previous models, so that new hypothesis and new agent behaviours could be tested.

In this paper, we replicate the model by Axtell et al. (2000) (hereafter AEY), where two agents demand a portion of the same pie, and the portion a particular agent gets depends on the portion demanded by the other agent. Our results are in agreement with their conclusions, both with non distinguishable and distinguishable agents (the tag model), as Dessalles et al. (2007) also confirmed in a previous replication of this work.

But we try to go one step further. First, we have considered possible artefacts (Galán et al., 2009) and we have tested the results to minor changes in the agents decision rule (as López-Paredes et al., 2004 suggested), so that their decision depends on the most likely option taken by their opponents in previous games; in particular, agents decide based on the opponents decision in a "statistical mode". It is consistent with experimental research done in neuroscience which demonstrates that humans don't use statistical properties in their internal decision processes.

And secondly, we have tested how dependent are the results of the reward values on the pay-off matrix, to see how it affects the aggregated observed behaviour.

The main result of our research is that these simple changes may affect dramatically how and when the equilibrium is reached. Our results confirm the important role of tags in the evolution of the system, which has been empirically demonstrated by Ito et al. (2007) that it plays a main role in 'rational' decisions.

2 Cognitive foundations

Social Neuroscience offers an opportunity to design more realistic agent based models. Lieberman (2007) stated that human beings have two systems that control the manner they behave in social situations. These systems are the X-system and the C-system. The X-system is responsible for social process that would be designated

as automatic and the C-system is responsible for social process that would be designated as controlled. In AEY's model, the agents take decisions at random with certain probability. This behaviour is a reflection of the decisions that one takes when his X-system is activated. On the other hand, the situations in which the agents take decisions in a rational way can be considered as an action taken by the C-system.

Ito et. al. (2007) state that human beings take different decisions depending on several cues (tags) such as the gender, race or age they distinguish in others (we create stereotypes to facilitate the decision process). Their work justify that these cues are the cause of prejudices and different behavioral paths in the relationship with others. In the AEY's model we study, the agents only distinguish a tag but there is not a differentiated rule of behaviour (there is not prejudice at all in the society). Agents use the same rule of decision, founded in past experience (in some way, looking to create a stereotype) but they save the record of the reward at each round (the opponent's choice) in a different memory set. The consequence is that segregation in the society emerges occasionally (even without prejudices). We find that this result is very interesting, and can explain the mechanism that drives the emergence of cluster in a wide range of economic problems: industrial districts, spatial monopolies, etc.

3 The model

We begin by replicating the bargaining model by AEY in which two players demand some portion of a pie. They could demand three possible portions: low, medium and high. As long as the sum of the two demands is not more than 100 percent of the pie, each player gets what he demands; otherwise each one gets nothing.

The authors assume a population of n agents that are randomly paired to play. Each agent has a memory in which he records the decision taken by his opponents in previous games. The agent uses the information stored in his memory to demand the portion of the pie that maximizes his benefit (with probability $1-\varepsilon$) and randomly (with probably ε).

At first, the authors assume that the agents are indistinguishable from one another, except for their memories about previous games. They conclude that, whenever there are not observable differences among the agents (the agents have not a distinguishable tag), there is only one possible state of equilibrium in which all the agents demand half of the pie. Otherwise, all the agents are either aggressive or passive (some of them demand low and some of them demand high), and no equilibrium is reached.

Secondly, the authors let the agents be distinguishable from one another by introducing a tag: they create two types of agents, each of whom with a different tag. The agents are capable of identifying their opponents' tag and they keep the portion of the pie demanded by their opponents in their memories, both with the same and different tag. In this case, the authors prove that, just by adding different tags to the players, discriminatory states can emerge under certain conditions, in which agents with different tags follow different behaviours.

4 The model with one agent type

4.1 Replication

First, we have replicated the AEY's model. We used the original payoff matrix (i.e. the combination of values for the different demands): 30 percent for low; 50 percent for medium and 70 percent for high. We also used the original decision rule.

 $\begin{array}{l} \underline{Problem \ approach} \\ \hline n \ - \ number \ of \ agents \quad \ensuremath{\varepsilon}\ - \ uncertainly \ parameter \\ m \ - \ memory \ length \ of \ each \ agent \\ S_i \ - \ space \ of \ agent's \ (\ i = 1, \ ..., \ n) \ possible \ strategies \\ \hline \rightarrow \ j \in [L, \ M, \ H] \ / \ M = 50, \ H = 100 \ - \ L, \ L \ < H \\ (L \ - \ select \ Low, \ M \ - \ select \ Medium, \ H \ - \ select \ High) \end{array}$

 $[v_1, v_2, ..., v_m]^i$ - memory array of agent *i*, which stores the strategies $v_k \in [L, M, H]$ chosen by the opponents in the *m* previous rounds

[A, B] - couple of agent randomly paired $/\frac{n}{2}$ randomly pairs by round If agent A chooses $i \in S_A$, and agent B chooses $j \in S_B$, they will receive $[i, j]if(i + j) \leq 100$, and [0, 0] if (i + j) > 100 (see Table 1, Combination of payoffs)

Decision rule

 n_j^A - number of positions with value $j \in [L, M, H]$ in the memory array of agent A $[v_1, v_2, ..., v_m]^A$

 $Pr(B_j^A) = \frac{n_j^A}{m}$ - Probability estimated by the agent A for the possibility that the opponent B selects the strategy *j* (equivalent to the relative frequency of occurrence of value *j* in the memory array of agent A)

The utility function for agent A when selects the strategy $i \in S_i = [L, M, H]$ is: $U(A_i) = i \cdot \sum_{j \in S_B} [Pr(B_j^A) \cdot V(i, j)]$ $/ i \in S_A; \quad V(i, j) = 1 \text{ if } (i + j) \leq 100; \quad V(i, j) = 0 \text{ if } (i + j) > 100$

Then, each agent A selects with probability $(1-\varepsilon)$ the strategy i that

maximizes its utility function: A select $i \in S_A = [L, M, H] / EU(A_i) = max U(A_i)$ And selects a random option $i \in S_A$ with probability ε . $\frac{Example}{n - 10; m - 5;}$ L = 30, M = 50, H = 70 $\Rightarrow S_A = [L, M, H] = [30, 50, 70] - space of possible strategies for agent A$ $if <math>[j_1, j_2, ..., j_m]^A = [30, 30, 50, 70, 30] - current memory array of agent A$ $n_{30}^A = 3, n_{50}^A = 1, n_{70}^A = 1 \Rightarrow Pr(B_{30}^A) = \frac{3}{5}, Pr(B_{50}^A) = \frac{1}{5}, Pr(B_{70}^A) = \frac{1}{5}$ $U(A_{30}) = 30 \cdot Pr(B_{30}^A) \cdot V(30, 30) + 30 \cdot Pr(B_{50}^A) \cdot V(30, 50) + 30$ $\cdot Pr(B_{70}^A) \cdot V(30, 70) = 30 \cdot \frac{3}{5} \cdot 1 + 30 \cdot \frac{1}{5} \cdot 1 + 30 \cdot \frac{1}{5} \cdot 1 = 30$ $U(A_{50}) = 50 \cdot Pr(B_{30}^A) \cdot V(50, 30) + 50 \cdot Pr(B_{50}^A) \cdot V(50, 50) + 50$ $\cdot Pr(B_{70}^A) \cdot V(50, 70) = 50 \cdot \frac{3}{5} \cdot 1 + 50 \cdot \frac{1}{5} \cdot 0 + 50 \cdot \frac{1}{5} \cdot 0 = 40$ $U(A_{70}) = 70 \cdot Pr(B_{30}^A) \cdot V(70, 30) + 70 \cdot Pr(B_{70}^A) \cdot V(70, 50) + 70$ $\cdot Pr(B_{70}^A) \cdot V(70, 70) = 70 \cdot \frac{3}{5} \cdot 1 + 50 \cdot \frac{1}{5} \cdot 0 + 50 \cdot \frac{1}{5} \cdot 0 = 42$ Agent A selects 70 with probability $(1 - \varepsilon)$, as it maximizes its utility function. $EU(A_{70}) = max U(A_i) = 42$ And selects a random option $i \in S_A = [30, 50, 70]$ with probability ε .

A simulation of this replication is shown in Figures 1 and 2. Both simulations were run with the same initial parameters (the same number of agents *-n*-, the same memory size *-m*-, and the same uncertainty parameter *-* ε -). Figure 1 shows an equitable equilibrium of the system after 100 iterations. Figure 2 shows a fractious state after 53 iterations.

After running a great number of simulations with the same parameters, we conclude that the probability of getting the fractious state (Figure 2) is very low in comparison with the probability of reaching an equitable equilibrium (Figure 1). The reason for this is that, in the long term, the benefit of choosing M becomes higher than choosing L or H, and thus, the agents tend to choose M, reinforcing the system tendency towards the equitable equilibrium. All the agents are initialized with random memories. Therefore, there is still a little chance that the initial values in the agents' memories lead to a fractious state like the one shown in **Figure 2**.

Both models, AEY's and our replication, produce the same result in relation with the time it takes for the system to reach an equitable equilibrium starting from random initial conditions: it increases as the memory size grows. This is plotted in **Figure 3**.

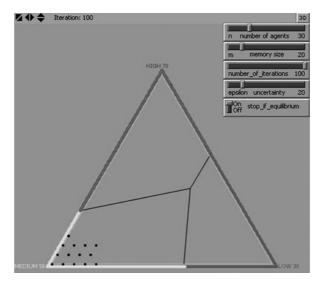


Fig. 1 Replication of AEY's model with with a number of agents n = 30, memory size m = 20 and uncertainty parameter $\varepsilon = 0.2$. Equitable equilibrium.

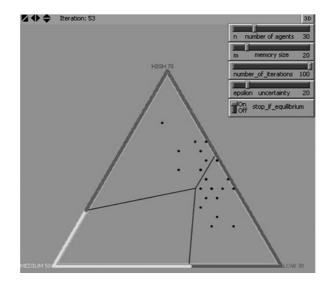


Fig. 2 Replication of AEY's model with with a number of agents n = 30, memory size m = 20 and uncertainty parameter $\varepsilon = 0.2$. Fractious state.

4.2 Introduction of a new decision rule

After replicating the original scenario, we changed AEY's decision rule so that the agents demanded the pie portion maximizing their benefits against the most likely option taken by their opponents in previous games (mode of their memory). An

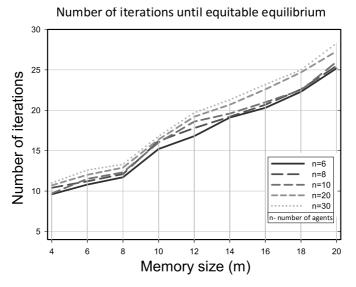


Fig. 3 Replication of AEY's model with uncertainty parameter $\varepsilon = 0$. Number of iterations to equitable equilibrium, as a function of the memory size; various *n* (number of agents).

agent will choose H if L is the most frequent decision taken by his opponents in the previous matches; If the most repeated value in his memory is M, the player will choose M. If previous matches show that H is the most frequent decision taken by his opponents, the agent will choose L.

New decision rule

Each agent A selects, with probability $(1-\varepsilon)$, its strategy i according to the statistical mode (Mo) of its memory array as follows:

$$\begin{split} Mo[v_1, v_2, ..., v_m]^A &= i \ / \ max \ n_j^A = n_i^A \ for \ all \ j \in S_A \\ If \ Mo[v_1, v_2, ..., v_m]^A &= L \ \Rightarrow A \ select \ i = H \\ If \ Mo[v_1, v_2, ..., v_m]^A &= M \ \Rightarrow A \ select \ i = M \\ If \ Mo[v_1, v_2, ..., v_m]^A &= H \ \Rightarrow A \ select \ i = L \\ And \ selects \ a \ random \ option \ i \in S_A \ with \ probability \ \varepsilon \ . \end{split}$$

$\frac{Example}{n = 10; m} = 5;$ L = 30, M = 50, H = 70 $\Rightarrow S_A = [L, M, H] = [30, 50, 70] - space of possible strategies for agent A$

 $if [v_1, v_2, ..., v_m]^A = [30, 30, 50, 70, 30]$ - current memory array of agent A

 $n_{30}^A = 3$, $n_{50}^A = 1$, $n_{70}^A = 1 \Rightarrow Mo[30, 30, 50, 70, 30] = 30 \Rightarrow Agent A selects 70$ with probability $(1 - \varepsilon)$, and selects a random option $i \in S_A = [30, 50, 70]$ with probability ε .

When the agents used this new decision rule, the chances of reaching the equitable equilibrium were considerably reduced (as López-Paredes et al., 2004 concluded). In fact, the probability of reaching the equitable equilibrium was not higher than reaching a fractious state.

Furthermore, even when the equity equilibrium was reached, the time to get it was longer in comparison with the same conditions in the experiment with AEY's decision rule. **Figure 4** and **Figure 5** show this comparison. Notice that the decision borders change after introducing the new decision rule.

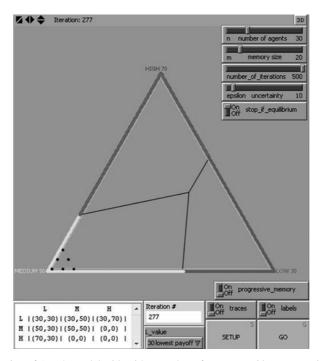


Fig. 4 Replication of AEY's model with with a number of agents n = 30, memory size m = 20 and uncertainty parameter $\varepsilon = 0.1$. Original decision rule.

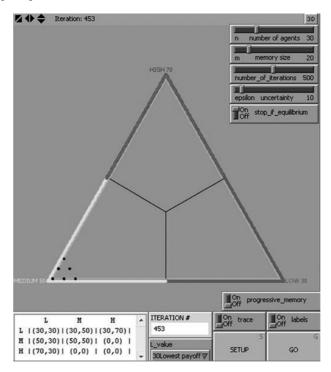


Fig. 5 Replication of AEY's model with with a number of agents n = 30, memory size m = 20 and uncertainty parameter $\varepsilon = 0.1$. New decision rule.

4.3 Introduction of a variable payoff matrix

In AEY's model, the values of the possible demands are fixed: 30 percent of the pie for low; 50 percent of the pie for medium and 70 percent of the pie for high. We have studied different combinations for low and high rewards to analyze the effects on the behaviour of the system¹.

The combination of payoffs is shown in Table 1.

The simulation shows that the higher the value assigned to "low", the longer it takes for the system to reach the equitable equilibrium. The reward an agent receives when he demands low is always L, independently of what his opponent demands. When the value assigned to L is increased, the agents are not given an incentive to choose M or H, because the reward of choosing one of these options gets lower than choosing L. Whereas the expected benefit of choosing L is fixed (L), the expected benefit of choosing M or H depends on the opponent's decision, which is conditioned by the values stored in his memory. This is the reason why in the first stages of the simulation, the agents tend to move towards the bottom-right corner of the simplex. Due to this behaviour, after a number of iterations, the appearance of L's

¹ In any case, the sum of the values of L and H is equal to the 100 percent of the pie.

	н	Μ	L		н	Μ	L		н	Μ	L
Η	0,0	0	95,5	н	0,0	0	80,20	н	0,0	0	65,35
Μ	0,0	50,50	50,5	Μ	0,0	50,50	50,20	Μ	0,0	50,50	50,35
L	5,95	5,50	5,5	L	20,80	20,50	20,20	L	35,65	35,50	35,35
	Н	Μ	L		Н	М	L		Н	М	L
Н	0,0	0	90,10	Н	0,0	0	75,25	Н	0,0	0	60,40
М	0,0	50,50	50,10	М	0,0	50,50	50,25	Μ	0,0	50,50	50,40
L	10,90	10,50	10,10	L	25,75	25,50	25,25	L	40,60	40,50	40,40
	H	Μ	L		Н	Μ	L		Н	Μ	L
Н	0,0	0	85,15	н	0,0	0	70,30	Н	0,0	0	55,45
Μ	0,0	50,50	50,15	Μ	0,0	50,50	50,30	Μ	0,0	50,50	50,45
L	15,85	15,50	15,15	L	30,70	30,50	30,30	L	45,55	45,50	45,45

Table 1 Possible payoff matrices (demand combinations).

in the agents' memories increases. Therefore, at some point, the expected benefit of choosing M becomes higher than the benefit of choosing L, and eventually all the agents choose M, reaching an equitable equilibrium. An analysis of this scenario is shown in **Figure 6**.

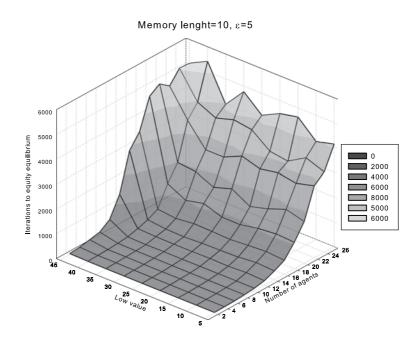


Fig. 6 Iterations to equitable equilibrium as a function of L (lowest payoff) and n (number of agents); uncertainty parameter $\varepsilon = 5$ and memory length m = 10.

5 The model with two agent types (the "tag" model)

In a second experiment, AEY attaches a tag to each agent, so that players are distinguishable from one another. There is only one tag in the model: the agent's colour. One half of the population will have a dark colour and the other half will have a light colour. The agents are capable of distinguishing the opponent's colour.

Although the decision rule does not change if the opponent has the same tag or not, the decision taken by the same-tag opponents are stored in a different memory set than the decisions taken by different-tag opponents.

AEY states that discrimination can emerge, both when the agents play with other agents of the same type (intra-type games) and when the agents play against players with different tag (inter-type). To analyze the results of the experiment, AEY uses two simplexes (**Figure 7**). The simplex on the left represents the memory of the agents when they play against players with the same tag (intratype matches). The simplex on the right shows the memory of the agents when they play against players with different tag (intertype matches).

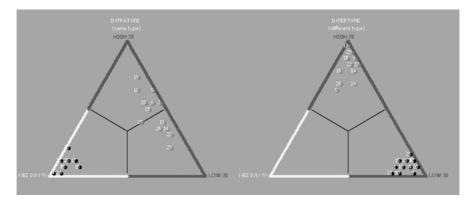


Fig. 7 Emergence of discrimination between players with different tags in our replication of AEY's model.

After simulating this scenario, we concluded that segregation did not appear in the model. This is why we tried changing the decision rule so that the agents chose the best reply against the most frequent decision taken by their opponents in previous matches (mode of the memory vector), as described in section 4.2. Once the decision rule was changed, we could appreciate all the cases of segregation that emerged in AEY's model, in both intratype and intertype games.

In **Figure 7**, segregation has emerged in both, intratype and intertype games. In the case of intratype games (matches among players with the same tag), the dark agents have learned to compromise and finally reach an equitable equilibrium. However, the light agents tend to choose L or H and reach a fractious state. In relationship with intertype games (matches among players with different tag), the system has reached a fractious state: the dark agents become aggressive and tend to demand H and the light players become passive and tend to demand L.

6 Conclusions

In AEY's model segregation processes emerge spontaneously. There is not a behaviour rule making agents behave in a different way when they play against agents with their same tag or with different tag. The only difference among the agents is the tag - which a priori does not need to influence on the decision as it is an external property - and the memories about the previous games.

Initially, the two types of agents are initialized with the same criteria to get a random memory. After a series of iterations with other agents, they "learn" how to behave depending on whether the agent they meet is same-tag opponent or a different-tag opponent.

After replicating the model we conclude that our results are in accordance with the original AEY's work. In this paper, trying to go a step further, we have inquired about the effects of new decision rules and new payoff matrix. We conclude that simple changes within the original model can produce dramatic changes in the studied system.

In future lines of research, we will include new decision rules, such as using moving averages when taking a decision and endorsement techniques to assign more relevance to the decisions taken in the recent games than in the older ones. We are currently working in playing the game in a 2D grid and with different social networks topologies, to study how the segregation can affect/be affected when agents are not randomly paired.

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